

The morning commute with heterogeneous trip length: on congestion and stability

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1 Extended abstract

The morning commute has historically been studied with point bottlenecks, following the seminal paper of Vickrey (1969). Urban networks however, cannot be modeled as collections of independent point bottlenecks. In fact, congestion propagates from one bottleneck to its neighbors, creating connected components of congestion that grow and may extend to the whole network (Ji et al., 2014). In the face of this complexity, an attractive solution consists in changing the modeling scale, from a point bottleneck to the network level (Small and Chu, 2003; Geroliminis and Levinson, 2009). This approach relies on empirically supported relationships (see e.g. Geroliminis and Daganzo (2008) and Buisson and Ladier (2009)), referred to as Macroscopic Fundamental Diagrams (MFD), which describe the dynamics of congestion under some homogeneity conditions with just a few variables such as accumulation, space-mean flow, trip completion rate and speed.

The consequences of this change of scale are, however, not fully understood yet. The first works combining an MFD with departure time choice called attention to the cost of hypercongestion, i.e. the phenomenon by which vehicle flow decreases with accumulation when accumulation exceeds a critical level (Small and Chu, 2003). Geroliminis and Levinson (2009) and Fosgerau and Small (2013) argued that by maintaining the system at the flow-maximizing accumulation, the benefits of congestion pricing with homogeneous users could be even greater than with Vickrey’s bottleneck, as the duration of the peak hour could be shortened. Arnott (2013) showed that further gains can be obtained by maintaining the accumulation always *below* its flow-maximizing value, at a level that increases with the peak duration. Yet, it is only very recently that Fosgerau (2015) and Daganzo and Lehe (2015) recognized how trip length heterogeneity challenges the fundamental First-In-First-Out assumption and started investigating its impacts on the morning commute.

Surprisingly however, Fosgerau (2015) and Daganzo and Lehe (2015) reached very different conclusions on the role of trip length. Using mathematically convenient but unconventional exponential-type scheduling preferences, Fosgerau (2015) showed that under some assumptions, the user equilibrium

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exhibits the so-called regular sorting property, i.e. two users differing only by their trip length sort according to a Last-In-First-Out (LIFO) pattern where the user with the longest trip starts earlier and finishes later. On the other hand, Daganzo and Lehe (2015) proved for the more conventional $\alpha - \beta - \gamma$ scheduling preferences but with a less realistic congestion mechanism (actually very similar to a point bottleneck) that the social optimum exhibits the First-In-First-Out (FIFO) property. Based on these theoretical considerations, Daganzo and Lehe (2015) also proposed a usage-based toll maintaining the accumulation near its flow-maximizing value and demonstrated numerically its benefits on the same realistic congestion mechanism as in Fosgerau (2015). The questions of the prevailing sorting pattern and of the optimal pricing strategy with heterogeneous trip lengths remain not fully solved, despite their crucial role in congestion management.

This work investigates the morning commute problem with inelastic demand and an MFD relating speed to accumulation. Our focus is primarily on the impact of trip length heterogeneity, but we also study the impact of heterogeneity in the scheduling preferences. In line with most of the literature, we rely on the so-called “fluid approximation” (Newell, 1982), in which the stochastic dynamics of a large number of agents are modeled by a deterministic real-valued process. With Vickrey’s constant capacity bottleneck, this assumption, together with some convexity assumptions on the schedule penalties, permitted to prove the existence and the uniqueness of an equilibrium distribution of arrival times (Smith, 1984a; Daganzo, 1985). In this paper, we leave aside the existence and uniqueness questions as they would require tedious derivations beyond the scope of the paper (even for a bottleneck model with much simpler dynamics, the proofs in Smith (1984a) and Daganzo (1985) contain a significant number of modeling assumptions, but also very long derivations). Instead, we obtain complementary results from analytical considerations and from simulations.

On the analytical side, it is shown for a large class of scheduling preferences that if users have continuously distributed characteristics, the network accumulation at equilibrium is a continuous function of time. With $\alpha - \beta - \gamma$ preferences and under certain conditions, a partial FIFO pattern emerges at equilibrium among early and late users. This FIFO pattern is strict only within families of users having heterogeneous trip lengths and identical preferences, or vice versa. Although we were not able to obtain a similar characterization for the social optimum, we show that introducing heterogeneous trip length may potentially lead to fully different results. It is shown in particular that the social optimum may, under specific circumstances, exhibit hypercongestion.

The simulation results provide several complementary insights. Various combinations of day-to-day learning and adjustment mechanisms were tested and all apparently converged (within some tolerance) for a wide range of demand intensities. Above a given threshold of demand intensity however, all mechanisms failed to converge. Instead, the system perpetually evolves within a small subregion of the feasible space. Such convergence difficulties were already reported in the literature for the constant capacity bottleneck model (de Palma, 2000) and were analytically explained by Iryo (2008) for an adjustment mechanism similar to the one described by Smith (1984b). While the specification of the day-to-day learning and adjustment behaviour may slightly impact the exact value of this threshold, a detailed analysis of private costs and externalities suggests that this instability is a characteristic of the dynamical system considered. Experiments with the various day-to-day learning and adjustment mechanisms suggest that this instability significantly increases the social cost. When properly implemented, congestion pricing can however bring the system back to a stable state, thus bringing yet another argument supporting its implementation. A more careful investigation of stability and

convergence for different congestion levels will be provided in the full paper.

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